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MULTIDISCIPLINARY OPTIMIZATION OF TRANSONIC WING DESIGN BASED ON EVOLUTIONARY ALGORITHMS COUPLED WITH CFD SOLVER

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Abstract. *Evolutionary Algorithms (EAs) were applied to multidisciplinary transonic wing design optimizations. Aerodynamic performances of the design candidates were evaluated by using the three-dimensional compressive Navier-Stokes equations to guarantee an accurate model of the flow field. The wing structure is modeled on a box-beam to estimate the wing thickness and wing weight. To overcome enormous computational time necessary for the optimization, the computation was parallelized on Numerical Wind Tunnel at NAL in Japan and NEC SX-4 computers at Computer Center of Tohoku University in Japan. First, a singleobjective wing design optimization was demonstrated by maximizing L/D with a structural constraint using a real-coded Adaptive Range Genetic Algorithm (ARGA). Because the structural constraint imposed a tradeoff between minimizations of the induced drag and the wave drag, the present ARGA found a compromised but reasonable design. Then, a multiobjective wing design optimization is performed by minimizing both drag and weight with a constraint on CL using a Multiobjective Evolutionary Algorithm (MOEA). Due to the tradeoff between minimization of aerodynamic drag and minimization of weight of wing structure, the solution to this problem is not a single point but a set of compromised designs. The present MOEA successfully captured these solutions that revealed the tradeoff information. These results showed that EAs were promising approach to multidisciplinary optimization problems.*

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1 INTRODUCTION

The design of wings is a typical example of multidisciplinary and multiobjective design optimization problems. Although the objective of a transonic wing design optimization is, in principle, minimization of aerodynamic drag, there are considerable number of tradeoffs. One of the main tradeoffs lies between minimization of drag and minimization of structural weight of wing. An increase in the wing thickness allows the same bending moment to be carried with reduced skin thickness, resulting in reduction of weight. On the other hand, it will lead to an increase in wave drag. In addition, an elliptical spanwise load distribution that minimizes induced drag results in a large bending moment at the inboard of the wing with an accompanying increase in weight. The solution to this problem is, therefore, not a unique optimal solution, but a set of compromised solutions, largely known as the tradeoff surface or Pareto-optimal solutions.

In general, the primary goal of multiobjective optimization problems (MOPs) is, unlike that of single objective optimizations, to find various Pareto-optimal solutions to show the precise tradeoff information among the completing objectives. Traditionally, solutions to MOPs are computed by the weighted-sum method that combines multiple objective functions into a scalar objective function. However, this method can find only one Pareto-optimal solution. In order to obtain various Pareto-optimal solutions, one has to optimize repeatedly with changing weights. This approach is inefficient and often fails to address tradeoffs.

Evolutionary Algorithms [1] (EAs) are particularly suited for MOP optimizations. By maintaining a population of solutions, they can uniformly sample various Pareto-optimal solutions in parallel without specifying weights between objectives. In addition, EAs have other advantages such as robustness, efficiency, as well as suitableness for parallel computing. EAs developed for MOPs are called Multiobjective Evolutionary Algorithms[1] (MOEAs).

In this paper, Evolutionary Algorithms are applied to multidisciplinary optimization of a transonic wing for generic transport aircraft. First, the transonic wing is optimized with respect to its aerodynamic performances using a single objective EA. Structural constraint is introduced to maintain the minimum wing thickness required to stand the bending moment due to the lift. The design result will reveal the nature of the tradeoff in a wing design problem. Then, multiobjective optimization of a transonic wing will be demonstrated using MOEA to minimize both drag and weight. The Pareto-solutions will identify the tradeoff information.

2 EVOLUTIONARY ALGORITHMS

EAs are emergent optimization algorithms mimicking mechanism of the natural evolution, where a biological population evolves over generations to adapt to an environment by selection, recombination and mutation. When EAs are applied to optimization problems, fitness, individual and genes usually correspond to an objective function value, a design candidate, and design variables, respectively. One of the key features of EAs is that they search from multiple points in the design space, instead of moving from a single point like gradient-based methods do. Furthermore, these methods work on function evaluations alone and do not require derivatives or gradients of the objective function. These features lead to the following advantages:

1) Robustness: EAs have capability of finding a global optimum, because they don't use function gradients that direct the search toward an exact local optimum. In addition, EAs have capability to handle any design problems that may involve non-differentiable objective function and/or a mix of continuous, discrete, and integer design parameters.

2) Suitability to parallel computing: Since EAs are population-based search algorithms, all design candidates in each generation can be evaluated in parallel by using the simple master-slave concept. Parallel efficiency is also very high, if objective function evaluations consume most of CPU time. Aerodynamic optimization using Computational Fluid Dynamics (CFD) is a typical case.

3) Simplicity in coupling CFD codes: As these methods use only objective function values of design candidates, EAs do not need substantial modification or sophisticated interface to the CFD code. If an all-out re-coding were required to every optimization problem, like the adjoint methods, extensive validation of the new code would be necessary every time. EAs can save such troubles.

Application of EAs to multiobjective design problems is also straightforward because EAs maintain a population of design candidates in parallel. This characteristic makes EA very attractive for solving MOPs. EAs developed for multiobjective optimization problems are called Multiobjective Evolutionary Algorithms (MOEAs). To solve MOPs successfully, the following two features are desired: 1) The solutions obtained are Pareto-optimal. 2) They are uniformly sampled from the Pareto-optimal set. To achieve these with EA, Pareto-based ranking method and fitness sharing technique are often used[2].

Owing to the above advantages over the analytical methods, EAs have become increasingly popular in a broad class of design problems[1]. EAs have been also successfully applied to aeronautical design problems including conceptual and preliminary design of aircraft[3,4], aerodynamic wing designs[5,6,7] and preliminary design of turbines[8].

3 SINGLEOBJECTIVE WING DESIGN OPTIMIZATION

3.1 Formulation of design problem

The objective of the present wing design problem is maximization of the lift-to-drag ratio *L/D* at the transonic cruise design point, maintaining the minimum wing thickness required to stand the bending moment due to the lift distribution. The cruising Mach number and the angle of attack are set to 0.8 and 0 degree, respectively.

The planform of the supercritical wing in the NASA Energy Efficient Transport (EET) Program[9] was selected as the test configuration for the following design cases (Fig.1). Wing profiles of design candidates are parameterized by the PARSEC airfoils[10]. A remarkable point is that this technique has been developed aiming to control important aerodynamic features effectively by selecting the design parameters based on the knowledge of transonic flows around an airfoil. It was reported that the PARSEC is the most efficient airfoil shape parameterization technique among typical parameterization techniques for aerodynamic optimization[11].

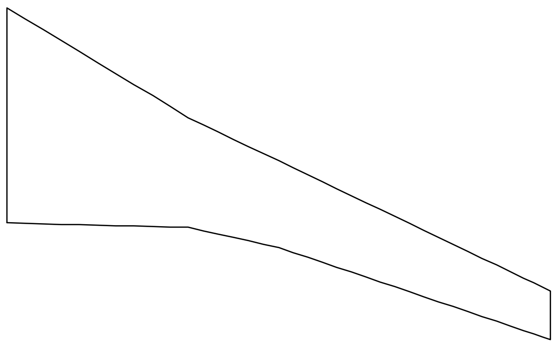

Figure 1: Wing planform

Similar to 4-digit NACA series airfoils, The PARSEC parameterizes upper and lower airfoil surfaces using polynomials in coordinates *X*, *Z* as,

$$Z = \sum_{n=1}^{6} a_n \cdot X^{n-1/2} \tag{1}$$

where a_n are real coefficients. Instead of taking these coefficients as design parameters, the PARSEC airfoils are defined by basic geometric parameters: leading-edge radius, upper and lower crest location including curvatures, trailing-edge ordinate, thickness, direction and wedge angle as shown in Fig. 2. These parameters can be expressed by the original coefficients a_n by solving simple simultaneous equations. Eleven design parameters are required for the PARSEC airfoils to define an airfoil shape in total. In this paper, ten design variables are used to give an airfoil shape with zero trailing-edge thickness.

The PARSEC parameters and the section angle of attack (in other words, root incident angle and twist angle) are given at seven span sections, of which spanwise locations are also treated as design variables except for the wing root and tip locations. The PARSEC parameters are rearranged from root to tip according to the airfoil thickness so that the resulting wings always have maximum thickness at the wing root. The twist angle parameter is also rearranged into numerical order from tip to root. The wing surface is then interpolated in spanwise direction by using the second-order Spline interpolation (Fig. 3). In total, 87 parameters determine a wing geometry.

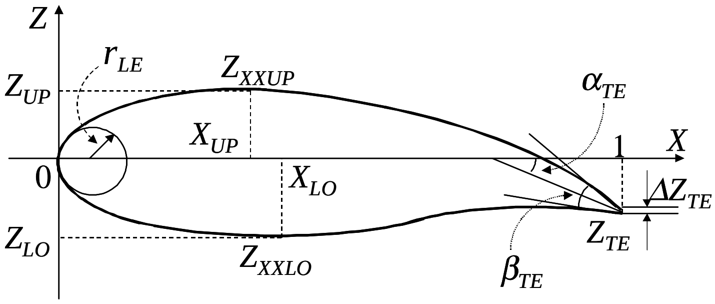

Figure 2: Design parameters for the PARSEC

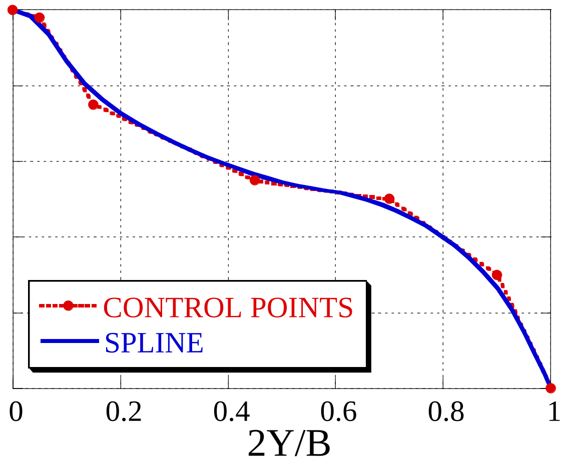

Figure 3: Spline interpolation

3.2 Aerodynamic analysis

The flow physics can be represented by a wide range of approximations. Among them, the Reynolds-averaged Navier-Stokes equations provide the state-of-aft of aerodynamic performance evaluation. Although a Navier-Stokes calculation requires large computer resources to estimate wing performances within a reasonable time, the three-dimensional Navier-Stokes equations must be solved because flows around a wing involve significant viscous effects, such as potential boundary-layer separations and shock wave/boundary layer interactions in the transonic regime. In this paper, a three-dimensional thin-layer Reynolds-averaged Navier-Stokes solver will be used to guarantee an accurate model of the flow field to demonstrate the feasibility of EA methodology. This code employs total variation diminishing type upwind differencing [12], the lower-upper symmetric Gauss-Seidel scheme [13], and the multigrid method[14].

3.3 Estimation of required thickness

To estimate the minimum thickness distribution to stand the bending moment due to the spanwise lift distribution, the wing structure is modeled by a thin walled box-beam as shown in Fig. 4. The skin panels of the box-beam are considered to shear the bending moment. From the load *L*, the spanwise bending moment distribution *M* is calculated by

$$\frac{d^2 M}{dy^2} = -L \tag{2}$$

For the brevity, the lift distribution is replaced by spanwise concentrated loads. The bending stress at each station is given by

$$\sigma = \frac{M}{I}\frac{t_1}{2} \tag{3}$$

where the second moment of area *I* is calculated as

$$I = 2 \cdot \left(\frac{t_1}{2}\right)^2 \cdot t_2 \cdot c = \frac{1}{2} t_1^2 \cdot c \cdot t_2 \tag{4}$$

The constraint is then given by the local stress to be less than the ultimate shear stress of, say, Aluminum alloy 2024-T351.

$$\sigma < \sigma_{ultimate} \tag{5}$$

Using Eqs. (3) to (5), we obtain the minimum thickness t_{min} at each segment,

$$t > \frac{M}{\sigma_{ultimate} \cdot c \cdot t_2} = t_{\min} \tag{6}$$

Following assumptions are made: the thickness of the skin panels is 2.5[cm] and its ultimate normal stress is 2.74×10^7[kgf/m^2]. The length of the chord at wing root C_{root} and maximum wingspan *b/2* are 10[m] and 18.8[m], respectively.

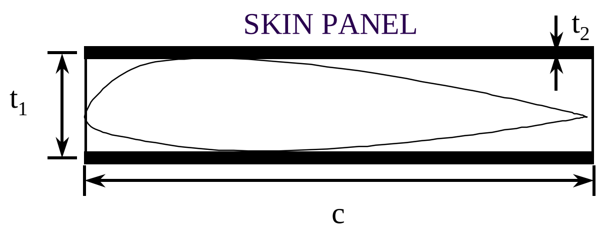

Figure 4: Box-beam modeling

3.4 Optimization

In this paper, real-coded Adaptive Range Genetic Algorithm (ARGA) [15] was used for the present singleobjective wing design optimization. The ARGA can solve large-scale design optimization problems very efficiently by promoting the population toward promising design regions during the optimization process.

The present EA adopts the elitist strategy[16] where the best and the second best individuals in each generation are transferred into the next generation without any recombination or mutation. The parental selection consists of the stochastic universal sampling[17] and the ranking method using Michalewicz's nonlinear function[18]. Blended crossover[19] (BLX-0.5) is used for recombination. Mutation takes place at a probability of 10% and then adds a random disturbance to the corresponding gene up to 10% of the given range of each design parameter. The population size is kept at 64 and the maximum number of generations is set to 65. The initial population is generated randomly over the entire design space.

The main concern related to the use of EAs coupled with three-dimensional Navier-Stokes solvers for aerodynamic shape designs is the required computational effort. In the present case, each CFD evaluation takes about 100 min. of CPU time even on a vector computer. Because the present optimization evaluates 64 x 65 = 4160 design candidates, sequential evolutions would take almost 7000 h (more than half a year!).

Fortunately, parallel vector computers are now available in many institutions and universities. In addition, EAs are intrinsically parallel algorithms and can be easily parallelized. One of such computers is *Numerical Wind Tunnel* (*NWT*) [20] located at National Aerospace Laboratory in Japan. NWT is a MIMD parallel computer with 166 vector-processing elements (PEs) and its total peak performance and the total main memory capacity are about 280 GFLOPS and 45GB, respectively. In the present optimization, evaluation process at each generation was parallelized using the master-slave concept; the grid generations and the flow calculations associated to the individuals of a generation were distributed into 64 PEs of NWT. This made the corresponding turnaround time almost 1/64 because the CPU time used for EA operators are negligible.

To handle the structural constraint with the single-objective EA, the constrained optimization problem was transformed into an unconstrained problem as

$$\boldsymbol{fitness\ \ function} = \begin{cases} 100 + \boldsymbol{L}/\boldsymbol{D} & \boldsymbol{if} \quad \boldsymbol{t} \geq \boldsymbol{t}_{\min} \\ (100 + \boldsymbol{L}/\boldsymbol{D}) \cdot \exp(\boldsymbol{t} - \boldsymbol{t}_{\min}) & \boldsymbol{otherwise} \end{cases} \tag{7}$$

where t and t_{min} are thickness and minimum thickness at the span station of the maximum local stress. The exponential term penalizes the infeasible solutions by reducing the fitness function value. Because some design candidates can have negative L/D, the summation of 100 and L/D is used.

3.5 Results

The optimization history of the present EA is shown in Fig. 5 in terms of L/D. During the initial phase of the optimization, some members had a strong shock wave or failed to satisfy the structural constraint. However they were weeded out from the population because of the resultant penalties to the fitness function. The final design has L/D of 18.91 satisfying the given structural constraint. Aerodynamic performances of the design are summarized in Table 1. Compared with a typical long-range transport aircraft, the present wing has smaller lift coefficient C_L. Although L/D is an important aircraft performance measure because the range of an aircraft depends on it, wing optimizations by maximizing L/D may result in a design that has too lower lift to fly. Therefore, another constraint on lift or multiobjective approach is required for a wing design optimization.

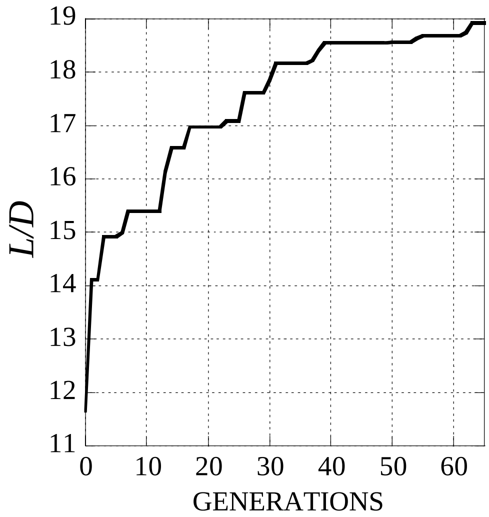

Figure 5: Optimization history of L/D

Table 1: Aerodynamic performances of the design

C_L	0.26213
C_D	0.01386
L/D	18.9143

The wing thickness distribution of the design is given in Fig. 6. The minimum thickness constraint appears at the kink because the inboard sections of the wing have large chord lengths and allow a large moment. The design satisfies this structural constraint while minimizing its thickness distribution to reduce the wave drag.

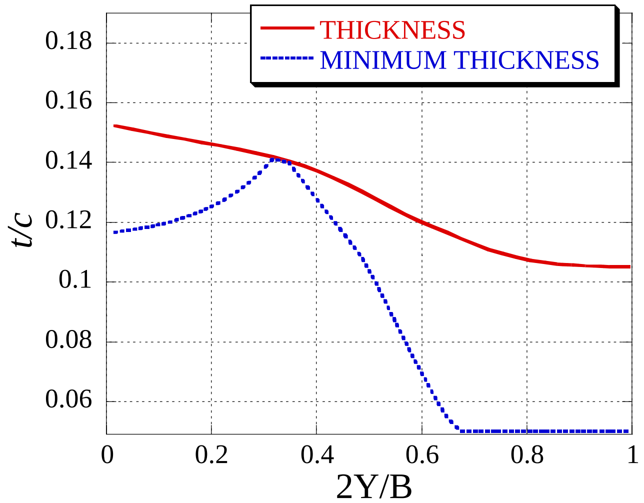

Figure 6: Spanwise thickness distribution

Figure 7 compares the span load distribution of the designed wing with a parabola that is known to give the minimum induced drag when the structural constraint is considered. The design does not have the parabolic span load distribution but a straight load distribution, which helps to reduce the bending moment at the inboard of the wing. The thickness distribution for the corresponding parabolic span load distribution is presented in Fig. 8. This figure indicates that a design that minimizes the induced drag would have 18% thickness-to-chord. Such design would result in an unacceptably large wave drag associated with a stronger shock wave. The present structural constraint imposed a tradeoff between minimizations of induced drag and wave drag. The present straight span load distribution is a compromised design.

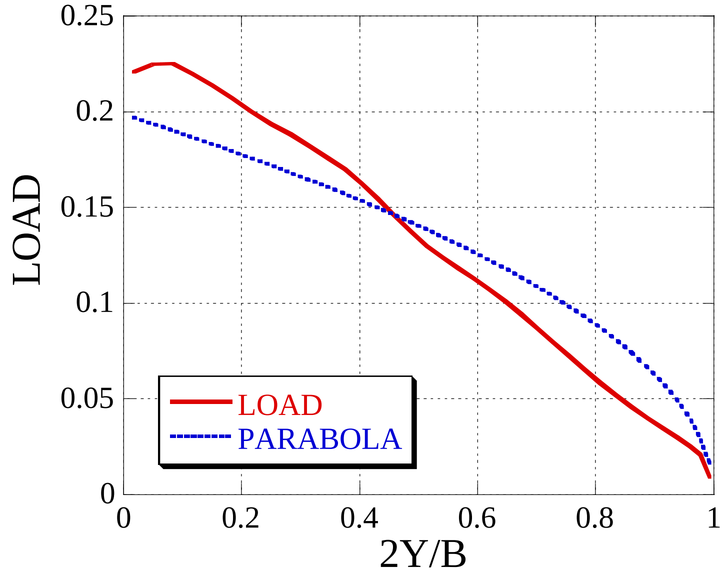

Figure 7: Spanwise lift distribution

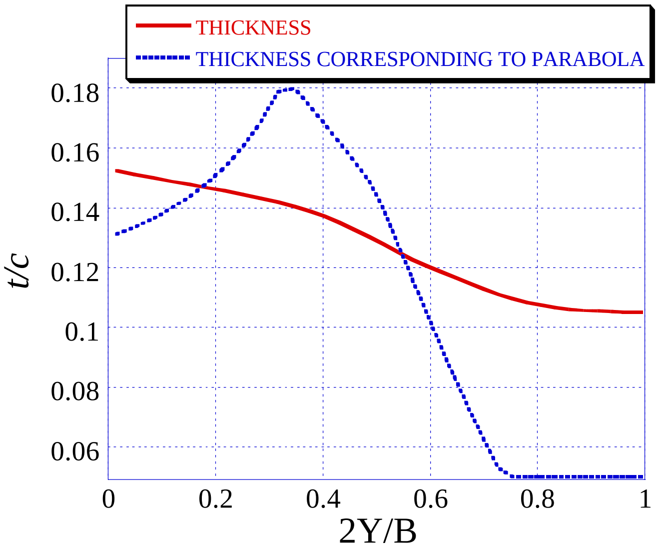

Figure 8: Comparison of thickness distributions between the present design and the minimum induced-drag design

The spanwise twist angle distribution and its control points are illustrated in Fig. 9. The angle of attack drastically decreases at the kink. Because the inboard of the wing has large chord length it allows large bending moment and thus large twist angle. On the other hand, since the outboard has smaller chord length, the wing requires significant twisting down outside the kink to reduce the moment.

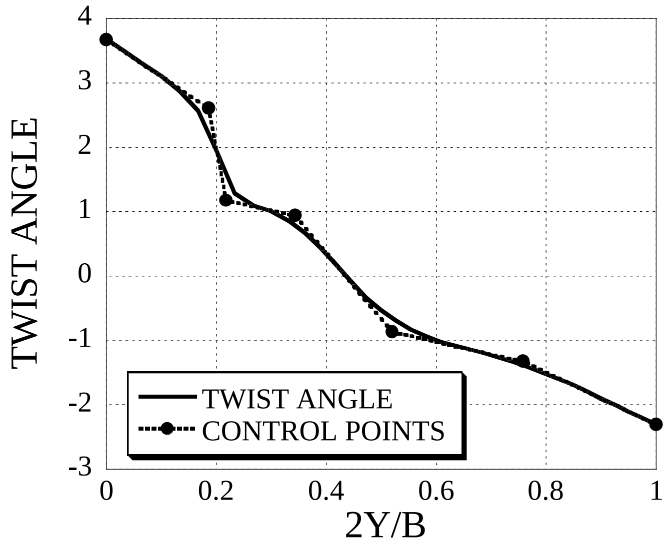

Figure 9: Spanwise twist angle distribution

The designed airfoil sections and the corresponding pressure distributions at the 0, 33, and 66% spanwise locations are shown in Fig. 10. Neither any strong shock wave nor any flow separation are found that may significantly increase pressure drag. This ensures the ability of the EA-based optimization in wing designs.

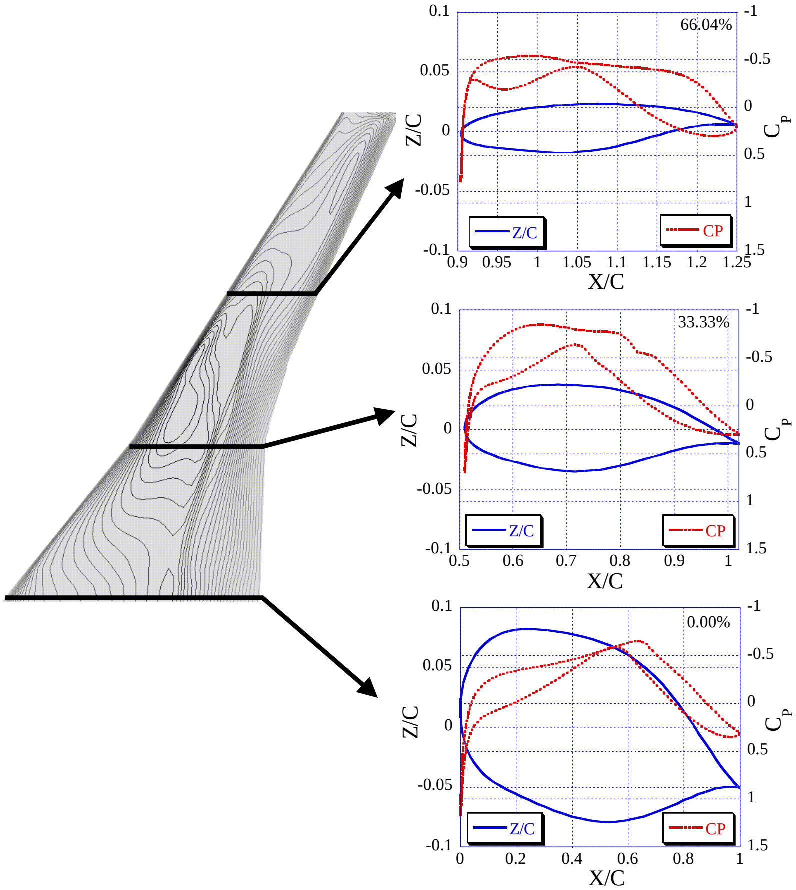

Figure 10: Designed airfoil sections and the corresponding pressure distributions

4 MULTIOBJECTIVE WING DESIGN OPTIMIZAITON

4.1 Formulation of design problem

The objective of the next design problem is multiobjective optimization of a transonic wing design, i.e., minimizations of both drag and weight with the constraint C_L=0.5. These objectives are competing and therefore the solution to this optimization problem is a set of compromised designs. MOEAs have capability to find those solutions in parallel. Flow conditions are same as the design in the previous section.

Constraints are usually enforced by a penalty function as the previous design case. However, such a penalty may reduce feasible design space. Therefore, the lift constraint is satisfied by changing the geometric angle of attack at wing root α_{root} so that C_L becomes 0.5 based on the lift coefficient varying linearly:

$$(\alpha_{root})_{CL=0.1} = \left(\frac{(C_L)_{SPECIFIED} - (C_L)_{\alpha=\alpha 1}}{(C_L)_{\alpha=\alpha 2} - (C_L)_{\alpha=\alpha 1}} \right) (\alpha_2 - \alpha_1) + \alpha_1 \tag{8}$$

where α_1 and α_2 are set to 3 and 6 degrees, respectively. This approach requires two extra flow evaluations.

Same Planform shape as the previous optimization with root chord of 12 [m] is used for this design. Wing profiles of design candidates are parameterized by the PARSEC airfoils. The PARSEC parameters and the section angle of attack are given at four span sections. The twist angle parameter is rearranged into numerical order from tip to root. The wing surface is interpolated in spanwise direction by using the second-order Spline interpolation. In total, 43 parameters determine a wing shape.

4.2 Aerodynamic analysis

Same CFD code described in subsection 3.2 is used.

4.3 Weight estimation

To estimate wing weight, a wing structure is modeled by box-beam structure consisting of upper/lower skin panels and front/rear spars as shown in Fig. 11. The skin panels shear the bending moment due to lift. By allowing 0.3% tensile, allowable minimum skin panel thickness is given by Eqs. (3),(4) as

$$t_2 = SC \frac{M}{\sigma_{allowed} \cdot c \cdot t_1} = SC \frac{M}{\varepsilon_{allowed} \cdot E \cdot c \cdot t_1} \tag{10}$$

where Young's modulus E=7.523 x10^9 [kg/ms^2] and $\varepsilon_{allowed}$ =0.003. Safety factor SC=1.5 is imposed. Once t_2 is obtained, the skin panel weight at each spanwise station is given by

$$W_{skin} = 2 \cdot \rho \cdot t_2 \cdot c \tag{11}$$

where density of Aluminum alloy 2024-T351 ρ=2.742 x10^7 [kg/ms^2].

Thickness of the front/rear spars is calculated by assuming the lift equals to the spar applied shearing stress τ at each spanwise station as

$$L = 2 \cdot \tau \cdot t_1 \cdot t_3 \tag{12}$$

Allowing 0.3% shearing strain, thickness of the front/rear spars t_3 is obtained as

$$t_3 = SC \frac{L}{2 \cdot \tau_{allowed} \cdot t_1} = SC \frac{L}{2 \cdot \gamma_{allowed} \cdot G \cdot t_1} \tag{13}$$

where modulus of transverse elasticity G=2.812x10^9[kg/ms^2] and $\gamma_{allowed}$ =0.003. Safety factor

SC=1.5 is imposed. The spar weight at each spanwise station is then given by

$$W_{spar} = 2 \cdot \rho \cdot t_3 \cdot t_1 \tag{14}$$

The total wing weight is finally calculated by summing skin panel and spar weight at all spanwise stations.

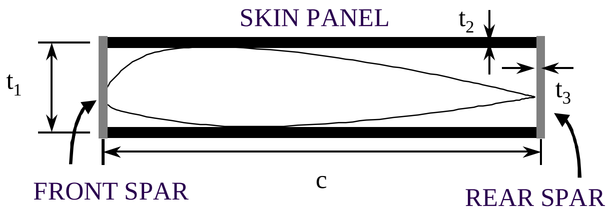

Figure 11 Box-beam modeling for weight estimation

4.4 Optimization

To find Pareto-optimal solutions, a real-coded MOEA is used. The present MOEA uses random parental selection and BLX-0.5. As the elitism, the best-N selection[21] is incorporated, where the best N individuals are selected for the next generation among N parents and N children based on Pareto-optimality[2] so that Pareto solutions will be kept once they are formed. A standard fitness sharing function[22] is used to maintain the diversity of the population. Since the strong elitism is used, high mutation rate of 0.2 is applied and a random disturbance is added to the parameter in the amount up to ±20% of the design space. Population size and maximum number of generations are set to 32 and 30, respectively. Unbiased initial population is generated by randomly spreading solutions over the entire design space in consideration. Evaluation is parallelized on NEC SX-4 computers at Computer Center of Tohoku University, using 32 PE's (this corresponds to one node of SX-4, a quarter of the center machine, and the node's peak performance is 64 GFLOPS with 8 GB memory).

4.5 Results

Pareto solutions obtained by the present optimization are shown in Fig. 12 with red points. The present MOEA successfully displayed the tradeoff information between minimization of drag and weight. Such tradeoff information is very helpful to a higher-level decision-maker in selecting a design with other considerations. In Ref [23], there is a description of the aerodynamic performance of B747, which is also plotted by a blue point though the planform shape is different to some extent. It is close to the tradeoff surface.

Figure 13 compares spanwise thickness distributions of the minimum drag design, the minimum weight design and a compromised design that has same C_D as B747. The minimum drag design minimizes the wave drag by reducing its wing thickness but on the contrary, requires the large structural weight. The minimum weight design has a very thick wing to reduce its weight but it leads to the large wave drag. The compromised design has a reasonable spanwise wing thickness distribution.

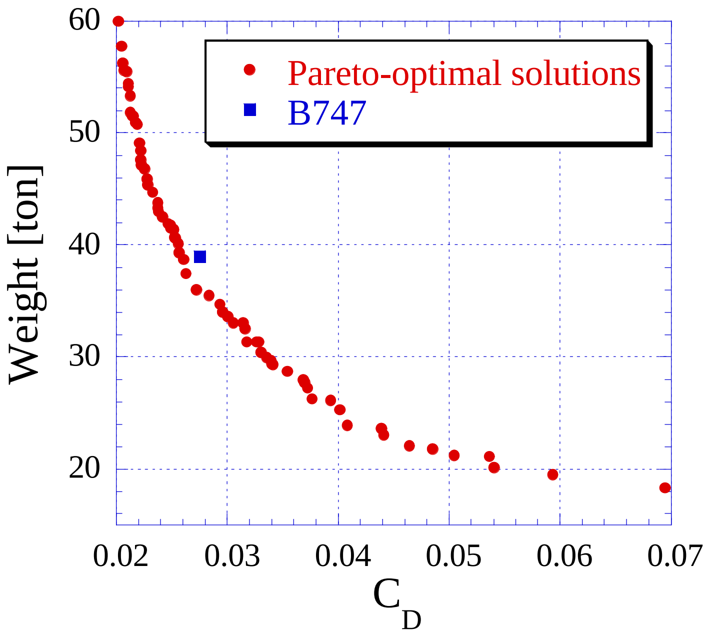

Figure 12 Pareto-optimal solutions in the objective function space

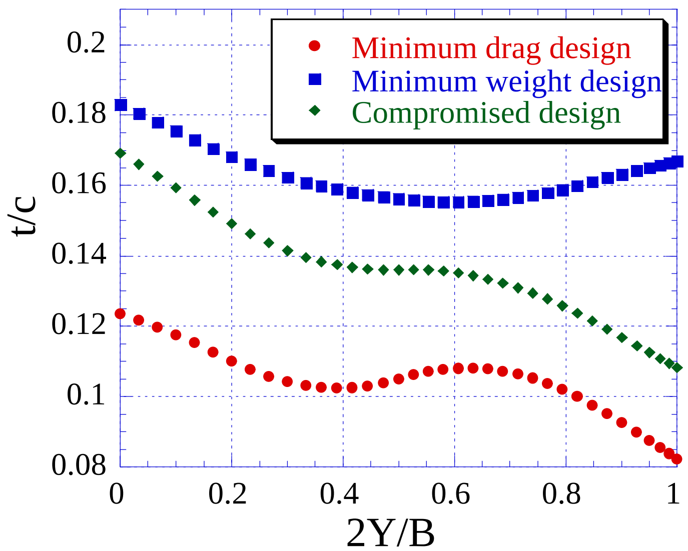

Figure 13 Comparison of spanwise thickness distributions

Figure 14 compares spanwise load distributions. As expected in the aerodynamic theory, the minimum drag design achieves the elliptical spanwise load distribution, which requires a heavy structure due to the large lift at the outboard of the wing. The minimum weight design, on the other hand, decreases lift at the outboard of the wing to reduce its moment, which, on the contrary, results in a substantial increase in induced drag. The compromised design has straight spanwise load distribution similar to the design obtained by the singleobjective optimization.

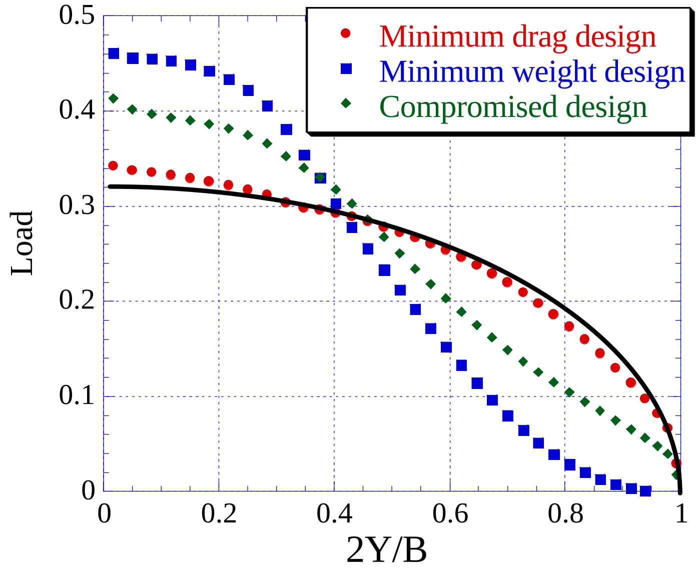

Figure 14 Comparison of spanwise load distributions

5 CONCLUSION

EAs were applied to singleobjective and multiobjective multidisciplinary wing optimizations. Aerodynamic performances of the design candidates were evaluated by using the three-dimensional compressive Navier-Stokes equations to guarantee an accurate model of the flow field. The required wing thickness and wing weight are estimated by modeling the wing structure on a box-beam. To overcome enormous computational time necessary for the optimization, the computation was parallelized on NWT and SX-4.

First, a single objective wing design optimization was demonstrated by maximizing L/D with a structural constraint using a real-coded ARGA. The designed wing has a good L/D value satisfying the given structural constraint on wing thickness. Because the structural constraint imposed a tradeoff between minimizations of the induced drag and the wave drag, the design did not have the minimum wave drag or the minimum induced drag. The straight span load distribution of the design was a compromise of this tradeoff.

Next, a multiobjective wing design optimization was performed by minimizing both drag and weight with a constraint on CL using a MOEA. Due to the tradeoff between minimization of aerodynamic drag and minimization of weight of wing structure, solutions to this problem become Pareto optimal. MOEAs are unique and attractive methods since MOEA finds many Pareto-optimal solutions in parallel. The present MOEA successfully captured these solutions that revealed the tradeoff information. These results show that EAs are promising approach to multidisciplinary optimization problems.

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