AN APPROACH FOR ROBUST DESIGN: MULTI-OBJECTIVE SIX SIGMA APPROACH

Akira Oyama[1*], Koji Shimoyama[2], and Kozo Fujii[3]

[1]Department of Space Transportation Engineering, JAXA Institute of Space and Astronautical Sciences, Japan – 3-1-1 Yoshinodai, Sagamihara, Kanagawa, 229-8510, Japan, oyama2@flab.isas.jaxa.jp; [2] Department of Aeronautics and Astronautics, University of Tokyo, currently, Tohoku University, Japan – shimoyama@edge.ifs.tohoku.ac.jp; [3]Department of Space Transportation Engineering, JAXA Institute of Space and Astronautical Sciences, Japan – fujii@flab.isas.jaxa.jp

Abstract - A new optimization approach for robust design, design for multi-objective six sigma (DFMOSS) has been developed and applied to a robust aerodynamic airfoil design for Mars exploratory airplane. The present robust aerodynamic airfoil design optimization using DFMOSS successfully showed the trade-off information between maximization and robustness improvement in aerodynamic performance in a single optimization run without careful input parameter tuning. The obtained trade-off information indicated that an airfoil with a smaller maximum camber improves robustness in terms of lift to drag ratio against the variation of flight Mach number.

Introduction

In real-world engineering designs, performance of a design may be very different from its expected value due to errors and uncertainties in design process, manufacturing process, and/or operating condition. A typical example of such critical situations is airplane wing design. It is well known that aerodynamic performance of an airplane is very sensitive to the wing shape and flight condition, and inevitable uncertainties such as wing manufacturing errors and wind variations may lead to drastic deterioration in aerodynamic performance of an airplane. In the airplane wing design, therefore, it is required not to use the conventional design optimization approach considering only optimality of performance at the design point, but to use the robust design optimization approach considering both optimality and robustness of performance against any uncertainties.

Objectives of this paper are to propose a new robust design optimization approach "design for multi-objective six sigma (DFMOSS)" by combining the ideas of the design for six sigma (DFSS) and multi-objective evolutionary algorithm (MOEA), and to carry out a robust aerodynamic airfoil design optimization for future Mars airplane by using the DFMOSS coupled with computational fluid dynamics (CFD) simulation.

Design for Multi-Objective Six Sigma

The idea of design for multi-objective six sigma (DFMOSS) is to incorporate MOEA into DFSS. In DFMOSS, the mean value μ_f and the standard deviation σ_f of the objective function $\boldsymbol{f(x)}$ are dealt with as multiple objective functions and thus minimized separately (for $\boldsymbol{f(x)}$ minimization problem) as follows:

$$\begin{aligned}&\text{Minimize: } \mu_f\\ &\text{Minimize: } \sigma_f\end{aligned} \qquad (1)$$

Figure 1 illustrates flowchart of robust optimization using DFMOSS. There is no need to pre-specify weighting factors before optimization as in DFSS, because DFMOSS deals with the multi-objective optimization problem. There is no need to pre-specify sigma level n either, because DFMOSS does not consider the constraint on sigma level n during the optimization process. The sigma level n satisfying the following conditions can be evaluated from the obtained robust optimal solutions in the post-process.

$$\begin{aligned}&\mu_f - n\sigma_f \geq LSL\\ &\mu_f + n\sigma_f \geq USL\end{aligned} \qquad (2)$$

where n represents user-specified sigma level, and LSL and USL are user-specified lower and upper objective function limits, respectively.

During the optimization process itself, multiple solutions (individuals) x_1, x_2, …, x_N are dealt with simultaneously using MOEA. For each individual, μ_{fi} and σ_{fi} are evaluated as two separate objective functions from $\boldsymbol{f(x)}$ at the sample points around x_i. Better solutions are selected based on the Pareto-optimality concept between μ_{fi} and σ_{fi}. Solutions for the next step are reproduced by crossover and mutation from the selected solutions. This optimization process is iterated until the trade-off relation between μ_f and σ_f has converged, and multiple robust optimal solutions are obtained.

The post-evaluation of sigma level n is illustrated in Fig.2, where four robust optimal solutions (A, B, C and D) obtained by a DFMOSS optimization were taken as example. The shaded region indicates the area satisfying the constraint of 6σ robustness quality. Solution B for instance, is included in the area satisfying the constraint of 3σ robustness quality, thus inferior to solution C in terms of robustness. Therefore, the satisfied sigma level of each obtained robust optimal solution can be evaluated in a flexible sense, considering the trade-off between optimality and robustness of design.

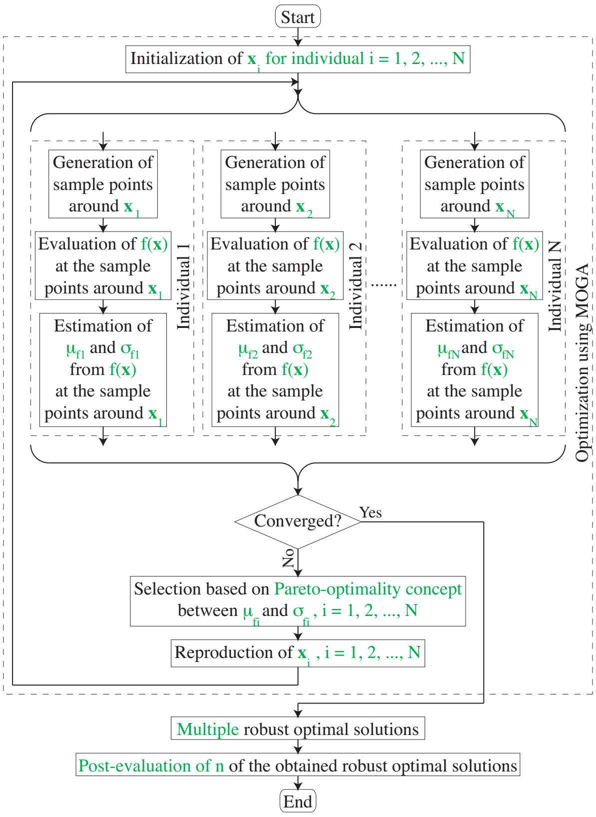

Figure 1 . Flowchart of DFMOSS

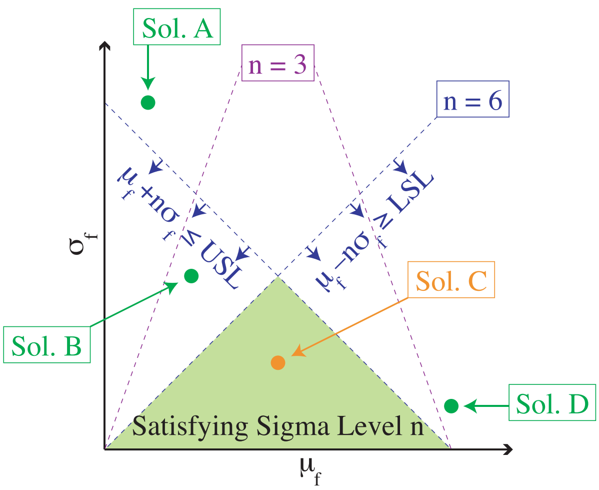

Figure 2. Post-evaluation of sigma level

Robust Aerodynamic Airfoil Design for Future Mars Airplane

Problem Definition

In this study, robust aerodynamic airfoil design optimizations against the variation of flight Mach number for a future Mars airplane are carried out. The cruising flight condition of NASA's "Airplane for Mars Exploration (AME)" [1] is adopted as the present design point; Reynolds number based on root chord length of 10^5, freestream Mach number M_{inf} of 0.4735, and the angle of attack of 2.0 degrees. It is assumed that freestream Mach number disperses around the design point (0.4735) in a normal distribution with a standard deviation of 0.1.

Airfoil configuration is defined by the B-spline curves with three fixed points corresponding to the leading and trailing edges and six control points whose coordinates can be specified flexibly, as shown in Fig.3 (here, c is the airfoil chord length). The design variables are chordwise x and vertical y coordinates of the six control points, therefore the number of design variables is twelve.

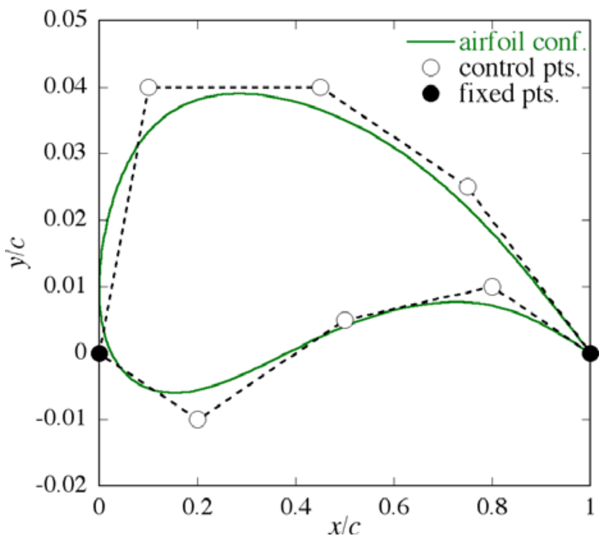

Figure 3. Airfoil shape parameterization

In this study, robustness of lift to drag ratio L/D is considered. When DFSS is used the objective function and constraint are defined as follows;

Minimize:
w_μ(mean value of L/D)+w_σ(variance of L/D)
Subject to:
(mean value of L/D) – n(standard deviation of L/D) > 42

When DFMOSS is used, the objective functions are defined as follows;

Minimize:
mean value of L/D
standard deviation of L/D

where no constraint is required for DFMOSS as sigma level can be defined in post-process.

Approach

Single-objective and multi-objective evolutionary algorithms are used for DFSS and DFMOSS approaches, respectively. The statistical values of objective function against dispersive design variables are estimated by the second-order Taylor's series expansion approach. Aerodynamic performance of an airfoil is evaluated by using a two-dimensional Navier-Stokes solver. For more detail of the optimization methods and the flow solver, see [2].

In the robust optimization using DFSS, the sigma level n is set to 3σ. Three optimization runs using DFSS with different weighting factors are performed.

Results

Figure 4 compares the robust optimal solutions obtained through DFSS and DFMOSS. The DFSS found three robust optimal solutions with more than 3σ robustness quality. However, these solutions distribute narrowly and sparsely. This indicates that the DFSS has lack in capability of revealing global trade-off relation between optimality (mean value of *L/D*) and robustness (standard deviation of *L/D*), and the DFSS requires more optimization runs with different combinations of weighting factors to obtain more detailed trade-off information. Fortunately, in the present optimizations using DFSS, three robust optimal solutions can be obtained because the pre-specified value of sigma level as 3σ is appropriate by chance. However, it is not always guaranteed for the DFSS to obtain the robust optimal solutions according to pre-specification of sigma level.

On the other hand, the DFMOSS found eighteen robust optimal solutions distributing globally and uniformly in the design space in a single optimization run. From this robust optimal solution distribution obtained through DFMOSS, global trade-off information between optimality and robustness can be understood easily; e.g., the maximum sigma level of *L/D* of the obtained solutions is more than 6σ by the post-evaluation when the lower specification limit of *L/D* is set to 42, and the standard deviation of *L/D* increases drastically when the mean value of *L/D* becomes larger than 44.5. In the present case, the robust optimization using DFSS found better robust optimal solutions than that using DFMOSS. This is because the DFMOSS searched an unexpectedly larger design space. However, such situation can be avoided easily by adding some constraints which eliminate unpractical design space.

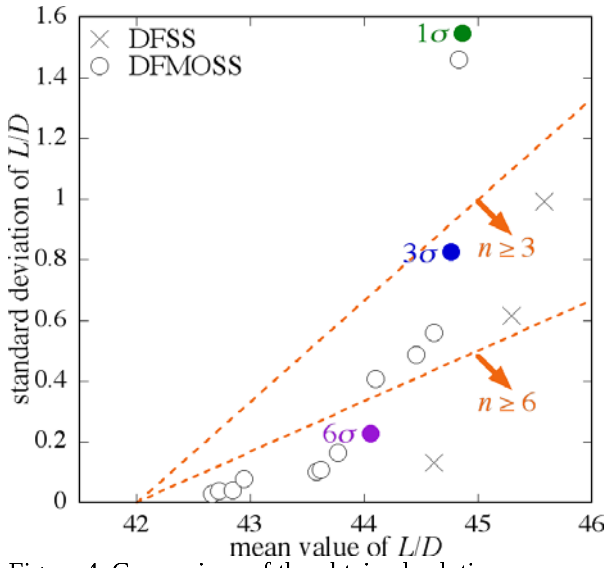

Figure 4. Comparison of the obtained solutions

Hereafter, three robust optimal solutions with 1σ, 3σ and 6σ robustness qualities of *L/D* obtained through DFMOSS (shown by closed circles in Fig.4) are compared and discussed. Figure 5 shows *L/D* of three robust optimal solutions against M_{inf}. In the robust optimal solution with 1σ robustness quality, *L/D* decreases drastically with an increment in M_{inf}, and it falls below its lower specification limit of 42 at high M_{inf}. On the other hand, the robust optimal solution with larger sigma level has slightly smaller *L/D* at the design point, but more stable characteristics keeping large *L/D* against the increment in M_{inf}. These results prove that the present robust aerodynamic design optimization using the DFMOSS actually found the multiple airfoil designs with various robustness qualities of *L/D* against the variation of M_{inf} by a single optimization run.

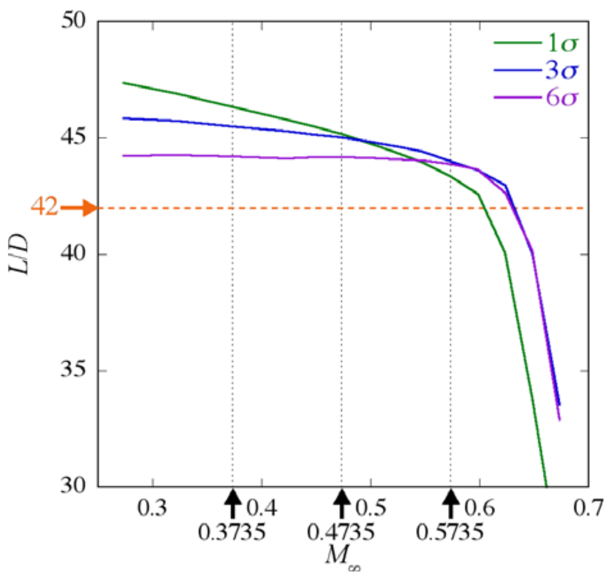

Figure 5. *L/D* of the three robust optimal solutions against M_{inf}

The airfoil configurations of these three robust optimal solutions in Fig. 6 show that maximum camber is one of the major trade-off factors between *L/D* and robustness improvements. The reason is that an airfoil with a smaller maximum camber realizes a smaller increment in pressure drag due to shock wave, and eventually improves the robustness in *L/D* against the increment in M_{inf}.

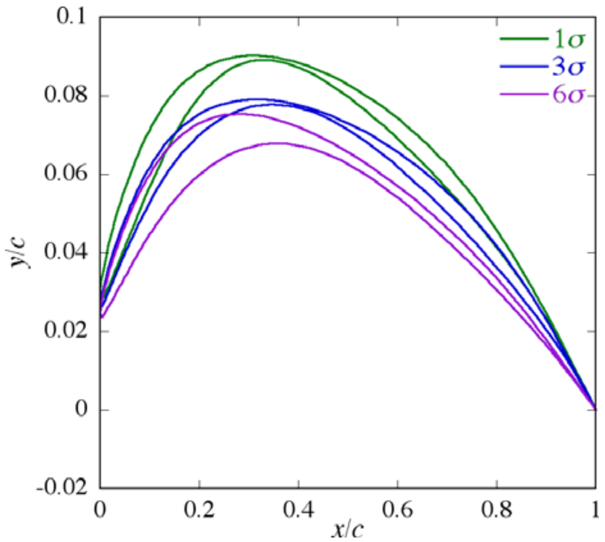

Figure 6. Airfoil configurations of the three robust optimal solutions

Concluding Remarks

In this paper, a new robust optimization approach called DFMOSS has been proposed by incorporating the idea of MOEA into DFSS, and the robust aerodynamic airfoil design optimizations for future Mars airplane have been carried out by using the DFMOSS coupled with the CFD simulation.

Compared to DFSS, the present robust optimizations using DFMOSS effectively revealed more detailed trade-off information between the optimality and the robustness of aerodynamic performances in a single optimization run without careful tuning of input parameters such as weighting factors and sigma level.

The robust airfoil design optimization using DFMOSS revealed that an airfoil with a smaller maximum camber improves its robustness in terms of lift to drag ratio against the variation of flight Mach number.

References

1. D. W. Hall, R. W. Rarks, and S. Morris; http://www.redpeace.org/final_report.pdf, 1997.
2. K. Shimoyama, A. Oyama, and K. Fujii; AIAA Paper 2007-1966, 2007.