Multi-Objective Optimization for Robust Airfoil Design Considering Design Errors and Uncertainties

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In real-world engineering designs, performance of a design may be very different from its expected value due to errors and uncertainties in design process, manufacturing process, and/or operating condition. A typical example of such critical situations is airplane wing design. It is well known that aerodynamic performance of an airplane is very sensitive to the wing shape and flight condition, and inevitable uncertainties such as wing manufacturing errors and wind variations may lead to drastic deterioration in aerodynamic performance of an airplane. In the airplane wing design, therefore, it is required to use not the conventional design optimization approach considering only optimality of performance at the design point, but the robust design optimization approach considering both optimality and robustness of performance against any uncertainties.

Improvements in optimality and robustness of performance are usually competing in real-world design problems. Therefore, there exist multiple compromised solutions (robust optimal solutions) between the optimality and the robustness. An objective of robust design optimization is to find these compromised solutions to reveal the trade-off information and at the same time, give the upper-level decision maker the option of selecting one solution from the compromised solutions with other consideration.

Therefore, in this paper, a new robust design optimization approach "design for multi-objective six sigma (DFMOSS)"[1] is proposed by combining the ideas of design for six sigma (DFSS)[2] and multi-objective evolutionary algorithm (MOEA)[3]. The DFSS is a popular robust optimization approach based on "six sigma concept", which is one of the management reform techniques aiming at establishment of business process with very small dispersion such that the range of $\pm 6\sigma$ (σ: standard deviation) around the mean value μ of performance parameter is included in the acceptable range. The level of dispersion can be defined as "sigma level n" satisfying the following constraints:

$$\begin{aligned} \mu - n\sigma &\geq \text{LSL} \\ \mu + n\sigma &\leq \text{USL} \end{aligned} \tag{1}$$

where LSL and USL are lower and upper specification limits, respectively. Larger sigma level indicates smaller dispersion, i.e., a more robust characteristic.

Consider a single-objective non-constrained optimization problem where the value of objective function $f(\boldsymbol{x})$ of design variables $\boldsymbol{x}$ must be minimized as follows:

$$\text{Minimize: } f(\boldsymbol{x}) \tag{2}$$

In the robust design optimization using DFMOSS, Eq. 2 is rewritten to the problem where the mean value μ_f and the standard deviation σ_f of the objective function $f(\boldsymbol{x})$ are dealt with as multiple objective functions and minimized separately as follows:

$$\begin{aligned} \text{Minimize: } &\mu_f \\ &\sigma_f \end{aligned} \tag{3}$$

Figure 1 illustrates the flowchart of robust optimization using DFMOSS. During optimization process, multiple solutions (individuals) $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N$ are dealt with simultaneously using MOEA. For each individual $i = 1, 2, \cdots, N$, μ_{f_i} and σ_{f_i} are evaluated as two separate objective functions from $f(\boldsymbol{x})$ at the sample points around $\boldsymbol{x}_i$. Better solutions are selected based on the Pareto-optimality concept between μ_{f_i} and σ_{f_i} for $i = 1, 2, \cdots, N$. Solutions $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_N$ in the next step are reproduced by crossover and mutation from the selected solutions. This optimization process is iterated until the trade-off relation between μ_f and σ_f is converged, and multiple robust optimal solutions are obtained. After the optimization, the sigma level n satisfying Eq. 1 is evaluated from the obtained robust optimal solutions. Figure 2 illustrates detail of post-evaluation of sigma level n. Now it is assumed that four robust optimal solutions (solution A, B, C and D) are obtained by a DFMOSS optimization. The shaded region indicates the area satisfying the constraint of 6σ robustness quality. Figure 2 indicates that solution C is included in the painted area, i.e., this solution has more than 6σ robustness quality. On the other hand, solutions A, B and D are not included in this painted area. However, solution B is included in the area satisfying the constraint of 3σ robustness quality, i.e., this solution has worse robustness quality of 3σ than solution C.

Next, in this paper, the robust aerodynamic airfoil design optimization is carried out by using the DFMOSS coupled with computational fluid dynamics (CFD) simulation to obtain practical airfoil design concept considering the robustness of aerodynamic performance. The present application is the airfoil design for Mars airplane[4] which is a new approach to explore Mars in spite of conventional orbiting satellites and rovers. The robust design optimization is much more required for the airfoil design of Mars airplane because the Martian atmosphere has very large wind variations leading to drastic deterioration in airfoil performance. Consider the following robust aerodynamic airfoil design optimization problem considering the robustness of lift to drag ratio L/D when flight Mach number M_∞ disperses around 0.4735 with its standard deviation of 0.1:

$$\begin{aligned} &\text{Maximize: mean value of } L/D \\ &\text{Minimize: standard deviation of } L/D \end{aligned} \tag{4}$$

An optimized airfoil configuration is defined by the B-spline curves. The present design variables are chordwise and vertical coordinates of six control points of the B-spline curves, therefore the number of design variables is twelve.

Figure 3 shows the robust optimal solution distribution (standard deviation of L/D against mean value of L/D) obtained by using the DFMOSS. The DFMOSS found multiple (total eighteen) robust optimal solutions distributing globally and uniformly in the design space successfully. From this robust optimal solution distribution, global trade-off information between optimality and robustness can be understood easily; *e.g.*, the maximum sigma level of L/D of the obtained solutions is more than 6σ by the post-evaluation when the lower specification limit of L/D is set to 42, and the standard deviation of L/D increases drastically when the mean value of L/D becomes larger than 44.5. Figure 4 compares the airfoil configurations of these three robust optimal solutions with 1σ, 3σ and 6σ robustness qualities obtained by using the DFMOSS. It indicates that maximum camber is one of the major trade-off factors between L/D and robustness improvements. The reason is that an airfoil with a smaller maximum camber realizes a smaller increment in pressure drag due to shock wave, and eventually improves the robustness in L/D against the increment in M_∞.

References

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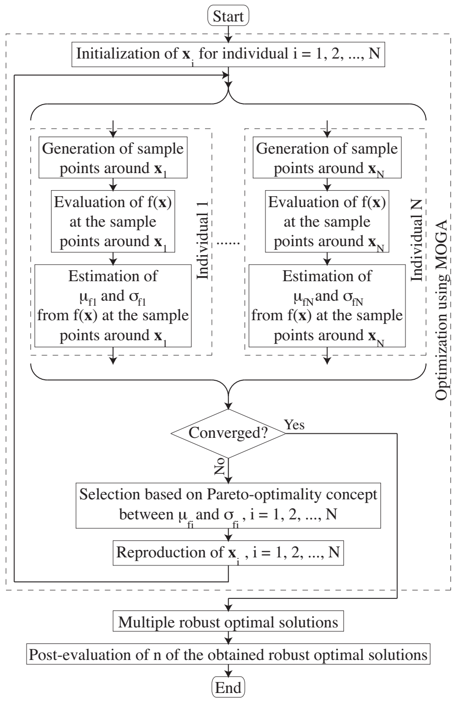

Figure 1: Flowchart of robust optimization using DFMOSS.

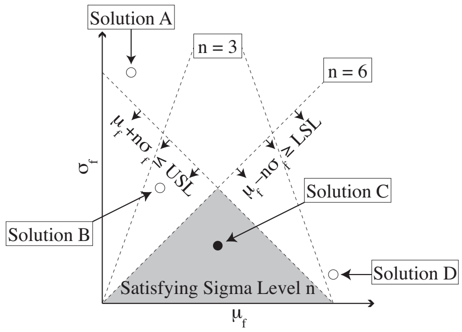

Figure 2: Post-evaluation of sigma level.

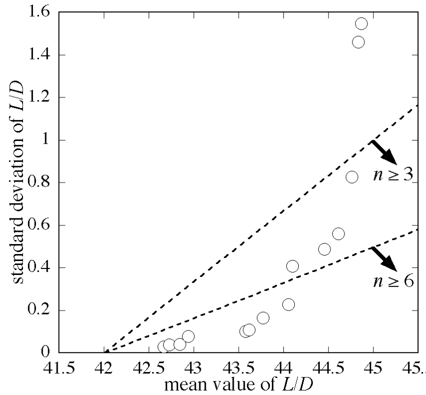

Figure 3: Robust optimal solutions.

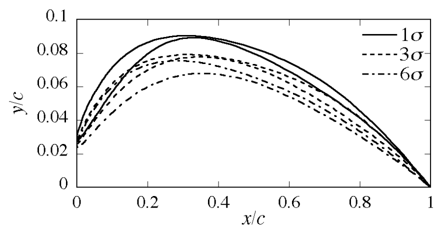

Figure 4: Airfoil configurations of three robust optimal solutions.